Homework Set 6

(sect 3.1 – 3.3)

For questions 1 and 2, compute the following determinants, using either cofactor expansion along the first row or the diagonal trick for computing the determinant for 3×3 matrices.

1. $\begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$

2. $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix}$

For questions 3 and 4, compute the following determinants by cofactor expansion. At each step, choose the row or column that will require the least number of computations.

3. $\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix}$

4. $\begin{vmatrix} 4 & 7 & 7 & 2 & 0 \\ 0 & 3 & 9 & 6 & -1 \\ 0 & 0 & 2 & 3 & 8 \\ 0 & 0 & 0 & 7 & 11 \\ 0 & 0 & 0 & 0 & -5 \end{vmatrix}$

5. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$. Write $3A$. Is $det(3A) = 3 \cdot det(A)$?

6. Find the determinant by row reducing to echelon form.

$$\begin{vmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{vmatrix}$$

7. Find the determinant by combining the methods of row reducing and cofactor expansion.

$$\begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{vmatrix}$$

8. Find the determinants below using the fact: $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$

a. $\begin{vmatrix} g & h & i \\ 2d & 2e & 2f \\ 3a & 3b & 3c \end{vmatrix}$

b. $\begin{vmatrix} a & b & c \\ d-2a & e-2b & f-2c \\ 5g & 5h & 5i \end{vmatrix}$

9. Let A and B be 4×4 matrices, with $det(A) = -1$ and $det(b) = 2$. Compute:

a. $det(AB)$

b. $det(B^5)$

c. $det(2A)$

d. $det(A^T A)$

e. $det(B^{-1}AB)$

10. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that $det(A+B) = det(A) + det(B)$ if and only if $a + d = 0$.

11. Use Cramer's Rule to compute the solution of the linear system.

$$\begin{cases} 2x_1 + x_2 + x_3 = 4 \\ -x_1 + 2x_3 = 2 \\ 3x_1 + x_2 + 3x_3 = -2 \end{cases}$$

12. Use Cramer's Rule to determine the values of the parameter s for which the linear system has a unique solution. Describe this solution(s).

$$\begin{cases} 3sx_1 - 5x_2 = 3 \\ 9x_1 + 5sx_2 = 2 \end{cases}$$

For questions 13 and 14, find the inverse of the given matrix by computing first computing the adjugate of the matrix.

13. $\begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

15. Find the volume of the parallelepiped with one vertex at the origin and the adjacent vertices at $(1,4,0)$, $(-2,-5,2)$, and $(-1,2,-1)$.